

Development of a Linear Stirling Model with Varying Heat Inputs

**Presented at the
5th International Energy Conversion Engineering Conference
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The linear model of the Stirling system developed by NASA Glenn Research Center (GRC) has been extended to include a user-specified heat input. Previously developed linear models were limited to the Stirling convertor and electrical load. They represented the thermodynamic cycle with pressure factors that remained constant. The numerical values of the pressure factors were generated by linearizing GRC's non-linear System Dynamic Model (SDM) of the convertor at a chosen operating point. The pressure factors were fixed for that operating point, thus, the model lost accuracy if a transition to a different operating point were simulated. Although the previous linear model was used in developing controllers that manipulated current, voltage, and piston position, it could not be used in the development of control algorithms that regulated hot-end temperature. This basic model was extended to include the thermal dynamics associated with a hot-end temperature that varies over time in response to external changes as well as to changes in the Stirling cycle. The linear model described herein includes not only dynamics of the piston, displacer, gas, and electrical circuit, but also the transient effects of the heater head thermal inertia. The linear version algebraically couples two separate linear dynamic models, one model of the Stirling convertor and one model of the thermal system, through the pressure factors. The thermal system model includes heat flow of heat transfer fluid, insulation loss, and temperature drops from the heat source to the Stirling convertor expansion space. The linear model was compared to a nonlinear model, and performance was very similar. The resulting linear model can be implemented in a variety of computing environments, and is suitable for analysis with classical and state space controls analysis techniques.

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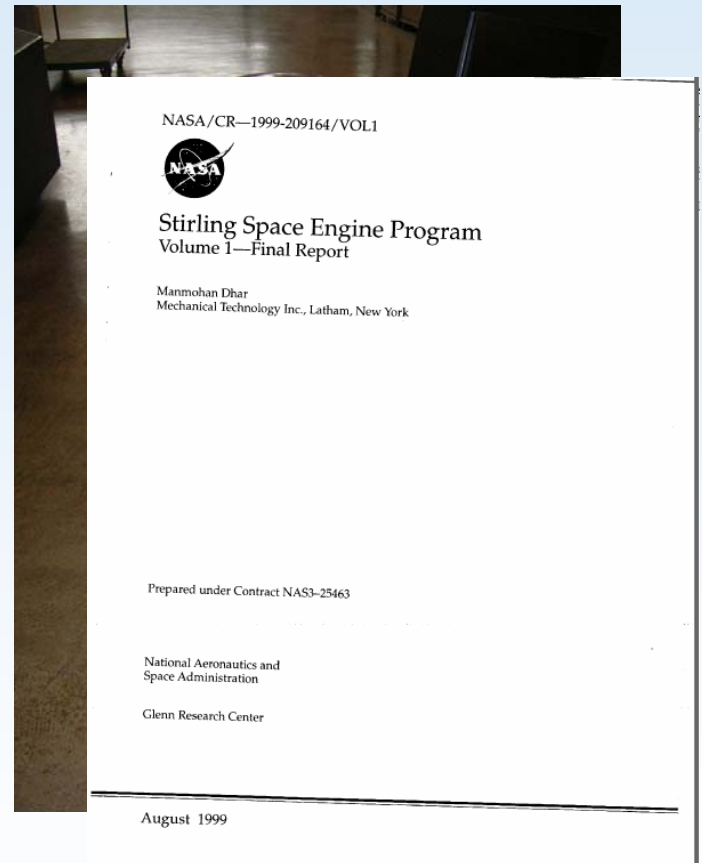
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Agenda

- System Dynamic Model (SDM) of CTPC
- Linear Model
 - Electro-mechanical system model
 - Thermal system model
 - Coupling the two
- Why a Linear Model
- Convertor Simulator

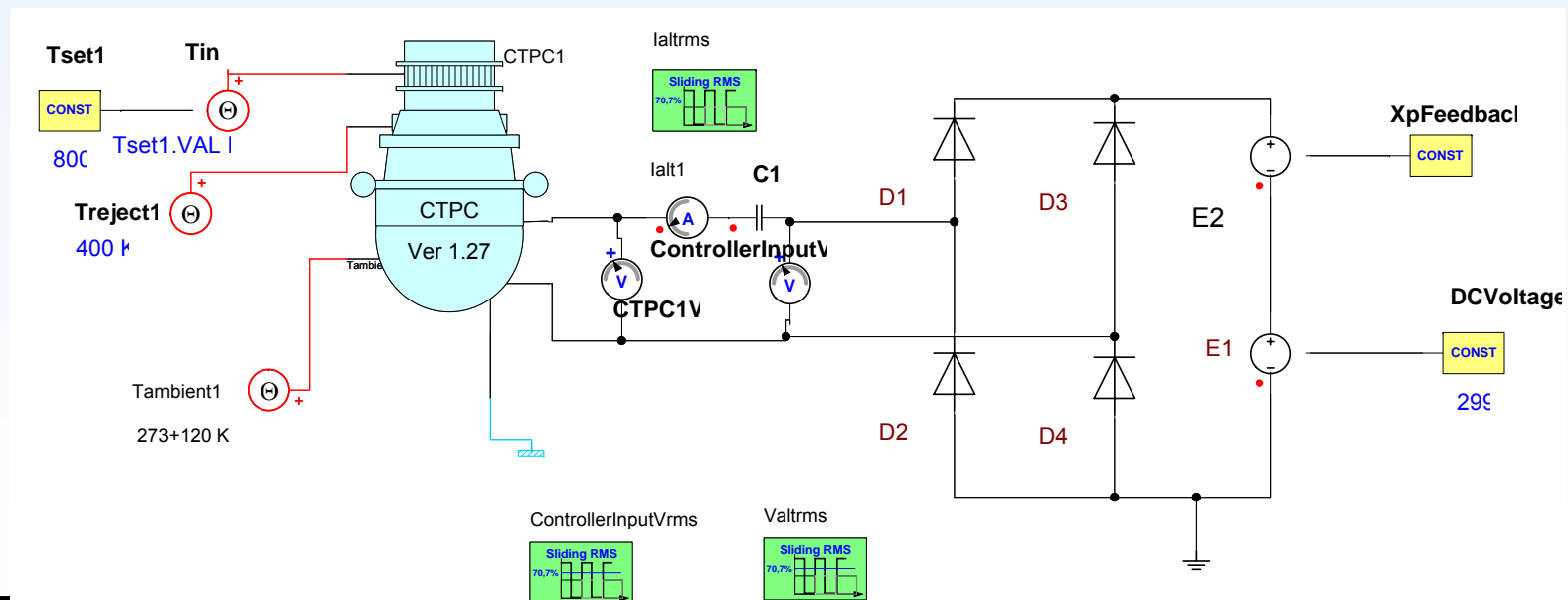
Component Test Power Convertor (CTPC)

- Built and tested in 1993
- Published report in public domain
 - Design Details
 - Test results
 - Has been used to validate Stirling convertor models.



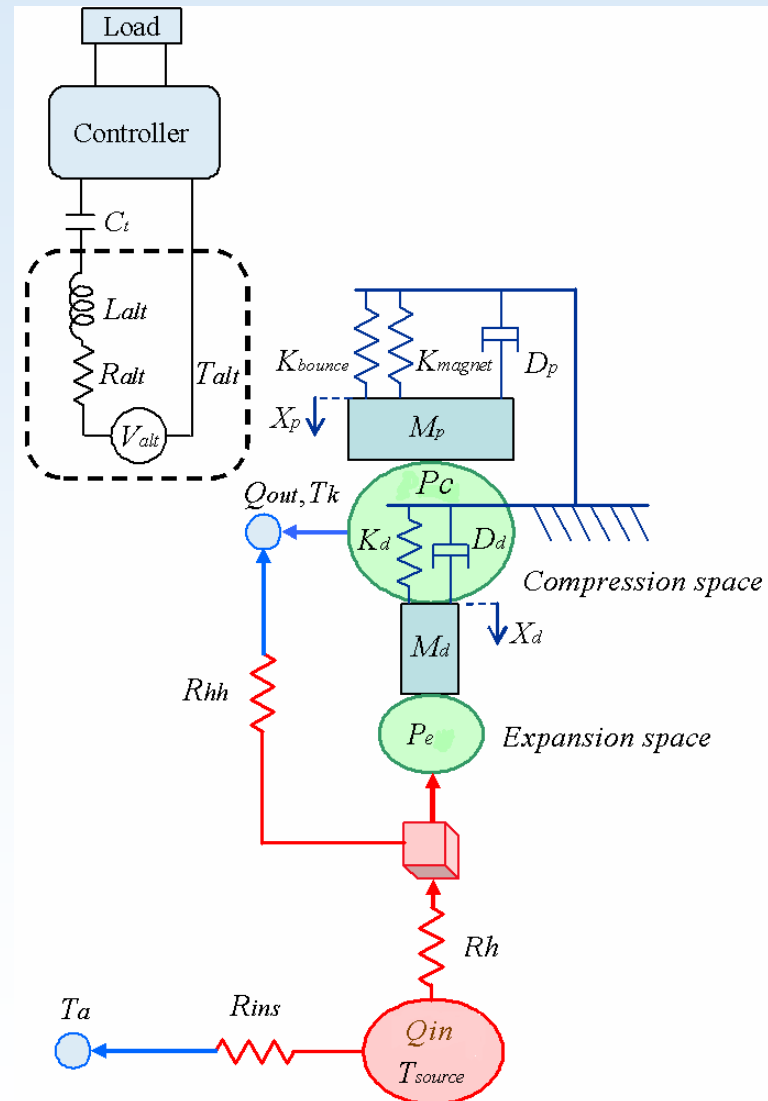
SDM Model of CTPC

- SDM model was assembled from published data
- SDM models the gas spring the regenerator flow friction with non-linear equations.



SDM Model of CTPC

- Electric
 - Alternator
 - Controller
 - Tuning Capacitor
- Mechanical
 - Piston
 - Displacer
 - Gas spring
 - Additional Damping
 - Ideal Gas Law
- Thermal
 - Insulation loss
 - Heater head conduction loss
 - Input heat or temperature
 - Rejection temperature



Linear CTPC Model

- Generic linear model of Stirling Convertor was described at 3rd IECEC in 2005.
- Developed models of
 - single convertor
 - dual opposed
 - single convertor with dynamic balancer
- However, the relation between output power and piston amplitude was not accurate.
 - No explicit thermal input
 - No provision was made for recalculating the pressure factors
- The need to include the differential equation to track the hot-end temperature
 - Necessary for the control system

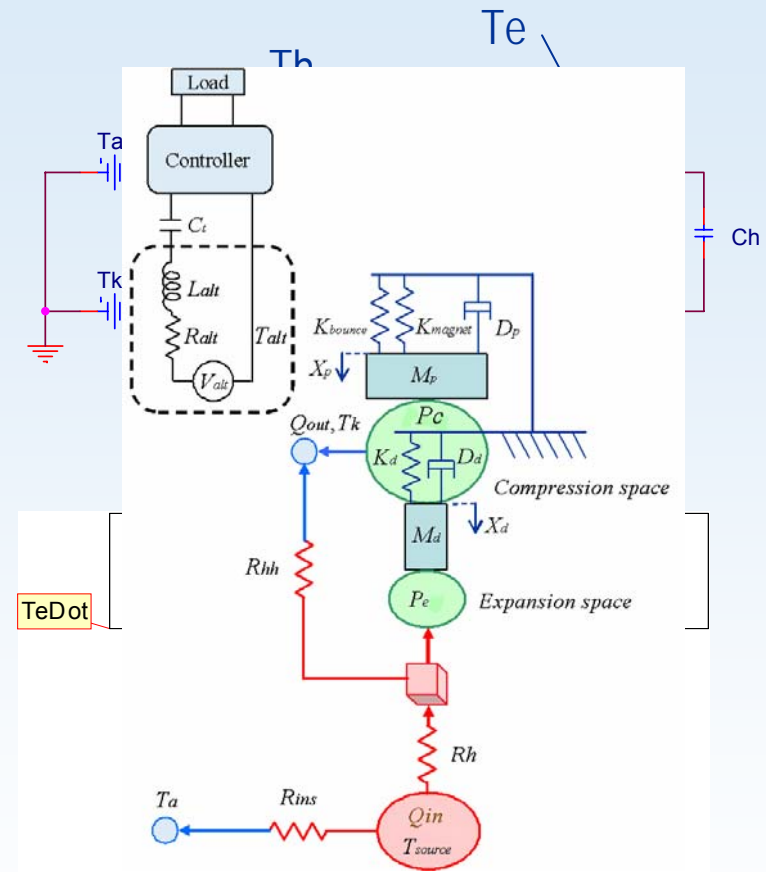
Electro-mechanical System

- Four state system x_d , \dot{x}_d , x_p , \dot{x}_p
- Input is alternator current
- Energy driving the system is represented by coefficients of x_p and \dot{x}_d .
- Energy damping the system is represented by coefficients of \dot{x}_p and \dot{x}_d as well as alternator current.

$$\begin{aligned}\frac{dx_d}{dt} &= \dot{x}_d \\ \frac{d\dot{x}_d}{dt} &= -\left(K_d + A_r \frac{\partial P}{\partial x_d}\right) \frac{1}{M_d} x_d + \left(A_d \frac{\partial \Delta P}{\partial \dot{x}_d}\right) \frac{1}{M_d} \dot{x}_d - A_r \frac{\partial P}{\partial x_p} \frac{1}{M_d} x_p + \left(A_d \frac{\partial \Delta P}{\partial \dot{x}_p}\right) \frac{1}{M_d} \dot{x}_p \\ \frac{dx_p}{dt} &= \dot{x}_p \\ \frac{d\dot{x}_p}{dt} &= -A_p \frac{\partial P}{\partial x_d} \frac{1}{M_p} x_d - \left(K_p + A_p \frac{\partial P}{\partial x_p}\right) \frac{1}{M_p} x_p - \frac{C_p}{M_p} \dot{x}_p + N \frac{d\Phi}{dx_p} \frac{1}{\eta_{mag}} \frac{1}{M_p} I_{alt}\end{aligned}$$

Thermal System

- Simplified thermal circuit was reduced to block diagram form.
 - Single integrator
- Block diagram implementation
 - Calculate expansion space temperature
 - Given Ambient Temp
 - Given Rejection Temp
 - Given Expansion Space PV power



Coupling the Two Systems

- Recalculation of mass of working fluid in the working space.
 - Temperature is not constant
 - Mean temperature changes
 - Working fluid goes from working space to bounce space on mean temperature increase.
 - Working fluid goes from bounce space to working spaces on mean temperature decrease

$$M_w = M_{tot} \cdot \frac{\left(\frac{V_h}{T_h} + \frac{V_r}{T_r} + \frac{V_k}{T_k} + \frac{V_{eo}}{T_e} + \frac{V_{co}}{T_c} \right)}{\left(\frac{V_h}{T_h} + \frac{V_r}{T_r} + \frac{V_k}{T_k} + \frac{V_{eo}}{T_e} + \frac{V_{co}}{T_c} + \frac{V_{bounce}}{T_{bounce}} \right)}$$

Coupling the Two Systems

- Pressure factors are used to couple the two
 - The pressure wave amplitude and phase are characterized by the pressure factors
 - Pressure factors are calculated from the ideal gas law.

$$P = M_w \cdot R_{gas} \cdot \left(\frac{V_h}{T_h} + \frac{V_r}{T_r} + \frac{V_k}{T_k} + \frac{V_e}{T_e} + \frac{V_c}{T_c} \right)^{-1}$$

$$\frac{\partial P}{\partial x_p} = \frac{A_p M_w R_{gas}}{(Q_e R_c + T_k)} \cdot \left[\frac{V_{co} - A_p x_p + (A_d - A_{rod}) X_d}{Q_e R_c + T_k} + \frac{V_k}{T_k} + \frac{V_r \ln(T_h / T_k)}{T_h - T_k} + \frac{V_h}{T_h} + \frac{V_{eo} - A_d x_d}{T_h - Q_e R_e} \right]^{-2}$$

$$\frac{\partial P}{\partial x_d} = -M_w R_{gas} \left[\frac{(A_d - A_{rod})}{Q_e R_c + T_k} - \frac{A_d}{(T_h - Q_e R_e)} \right] \cdot \left[\frac{V_{co} - A_p x_p + (A_d - A_{rod}) X_d}{Q_e R_c + T_k} + \frac{V_k}{T_k} + \frac{V_r \ln(T_h / T_k)}{T_h - T_k} + \frac{V_h}{T_h} + \frac{V_{eo} - A_d x_d}{T_h - Q_e R_e} \right]^{-2}$$

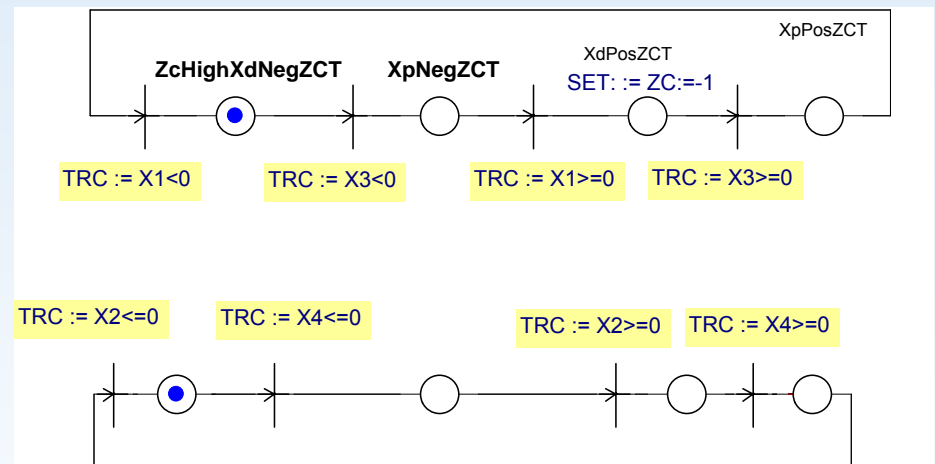
- When calculated in this way the values vary over the cycle.

Coupling the Two Systems

- Uses zero crossings of the piston and displacer velocity to find piston and displacer position amplitude.
- Re-calculates the pressure factors once per cycle
 - Piston pressure factor at displacer position zero crossing
 - Displacer pressure factor at piston position zero crossing
- Recalculates Expansion space PV power once per cycle

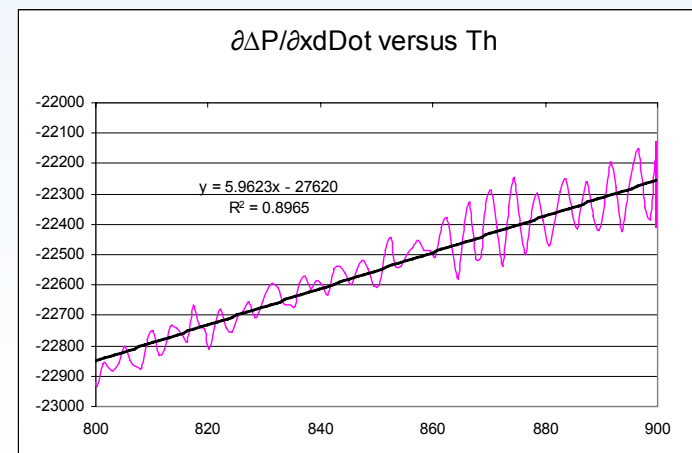
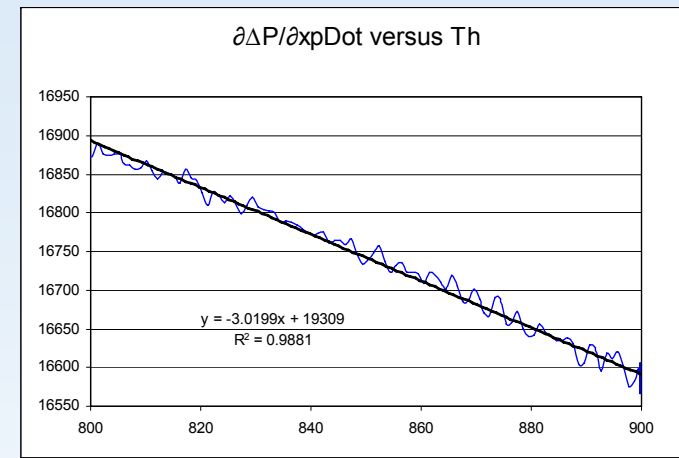
$$Q_{pv} = \pi A_d f X_d X_p P_p \sin \phi_d$$

State diagrams aid computation



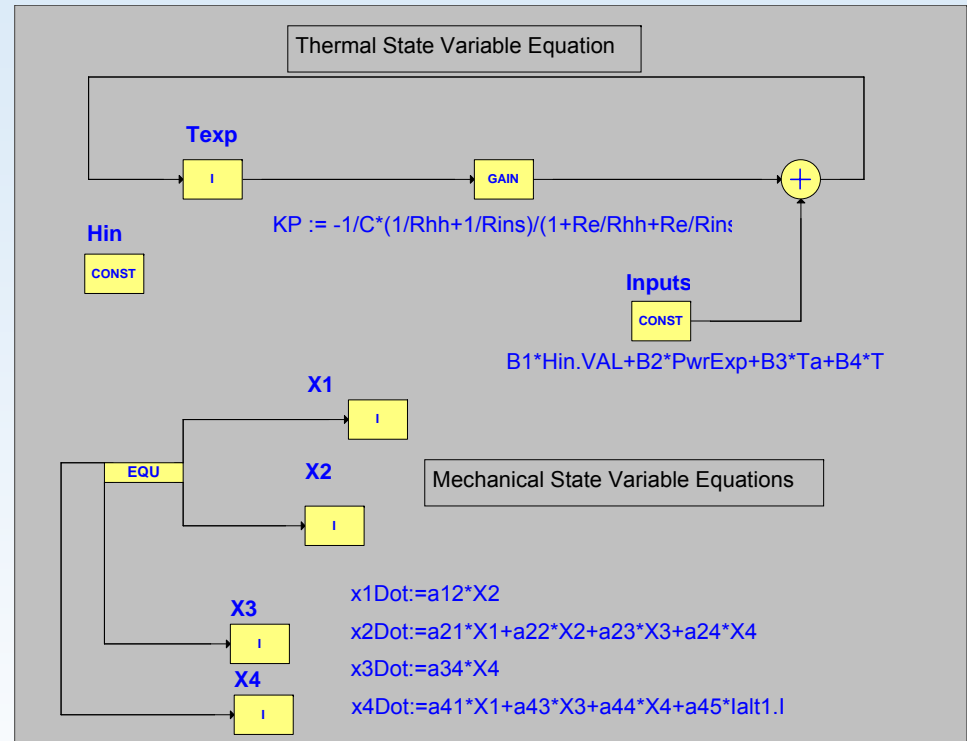
Pressure-drop Factors

- Pressure drop across the regenerator and the heat exchangers produces the damping for the displacer.
- The non-linear functions modeling the pressure drop are modeled in SDM
- A temperature sweep in SDM recorded the value of the pressure drop factors.
- Pressure drop is re-created when the factors are multiplied by piston velocity and displacer velocity.



Linear CTPC Model

- The 4-state mechanical system is unchanged from the 2005 single convertor version.
- The single state Thermal State-variable Equation uses expansion space temperature as the state variable.
- Four thermal inputs are distinguished
 - Qin Input heat
 - Qpv Expansion space PV power
 - Ta Ambient temperature
 - Tk Rejection temperature

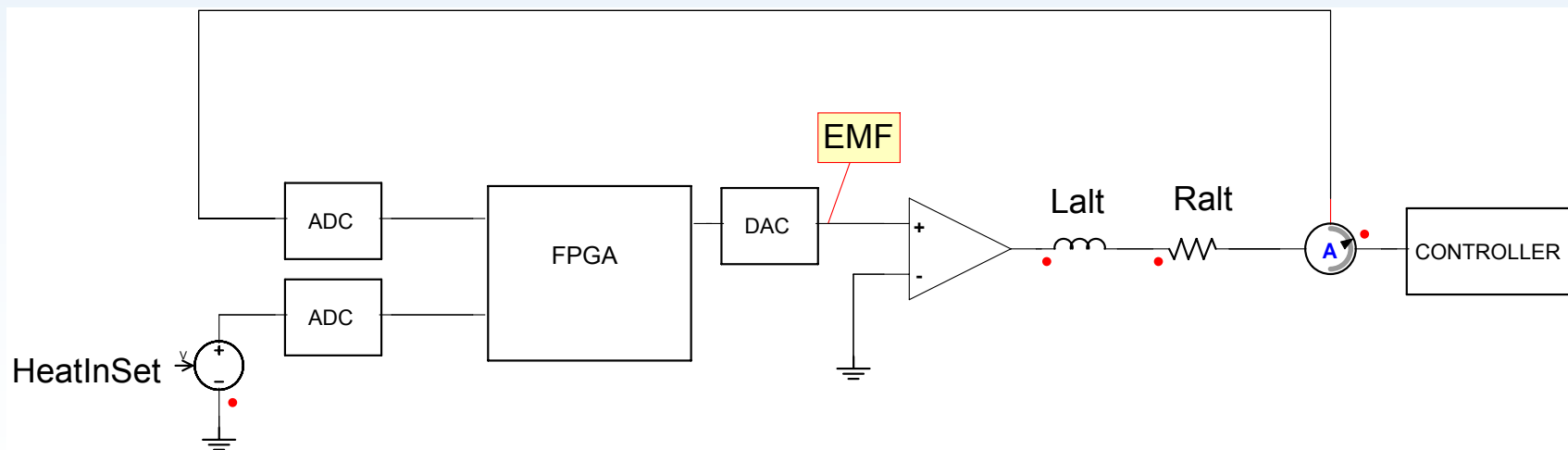


Why a Linear Model

- Linear model can be executed in real time

Stirling Convertor Simulator

- Implement the linear model in FPGA or microprocessor
- Provide model input values as inputs to the system
- Model output piston velocity is amplified by alternator constant to equal the alternator EMF.
- EMF is wired through L_{alt} and R_{alt} to equal convertor terminal voltage
- Controller hardware is attached.
- Measured current provides the current input to the model



Summary

- A four-state, single-input, single-output state space model of the piston and displacer motion and damping
- A single-state, four-input, single-output state space model of the thermal processes in the hot end.
- Re-calculation of the mass of the working fluid in the working spaces as shown.
- Re-calculation of the piston pressure factor and associated A-matrix entries, a_{23} and a_{43} whenever a change in either temperature, amplitude or phase angle is detected.
- Re-calculation of the displacer pressure factor and associated A-matrix entries, a_{21} and a_{41} whenever a change in either temperature, amplitude or phase angle is detected.
- Re-calculation of the pressure drop factor for displacer damping with respect to displacer velocity from and the associated A-matrix entry a_{22} .
- Re-calculation of the pressure drop factor for displacer damping with respect to piston velocity from the associated A-matrix entry a_{24} .

Conclusion

- With addition of the thermal differential equation and the recalculation of the pressure and pressure drop factors, the linear model can accurately track temperature changes due to disturbances in the load or in the thermal environment.
- Such a model can be used to exercise control algorithms that increase stroke to limit hot-end temperature.

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